

BCS pairing in a trapped dipolar Fermi gas.

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We present a detailed study of the BCS pairing transition in a trapped polarized dipolar Fermi gas. In the case of a shallow nearly spherical trap, we find the decrease of the transition temperature as a function of the trap aspect ratio and predict the existence of the optimal trap geometry. The latter corresponds to the highest critical temperature of the BCS transition for a given number of particles. We also derive the phase diagram for an ultracold trapped dipolar Fermi gases in the situation, where the trap frequencies can be of the order of the critical temperature of the BCS transition in the homogeneous case, and find the critical value of the dipole-dipole interaction energy, below which the BCS transition ceases to exist. The critical dipole strength is obtained as a function of the trap aspect ratio. Alternatively, for a given dipole strength there is a critical value of the trap anisotropy for the BCS state to appear. The order parameter calculated at criticality, exhibits novel non-monotonic behavior resulted from the combined effect of the confining potential and anisotropic character of the interparticle dipole-dipole interaction.

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I. INTRODUCTION

One of the most challenging goals of modern atomic, molecular and optical physics is to observe the superfluid (BCS) transition [1] in trapped Fermi gases. The possibility of such transition for gases with attractive short range interactions has been predicted in Refs. [2], and has been a subject of very intensive experimental investigations since then (for the latest experimental results see [3]). In typical experiments evaporative cooling is used to cool fermions. However, since the Pauli principle forbids the s -wave scattering for fermions in the same internal state, Fermi-Fermi [4] or Fermi-Bose [5] mixtures have to be used to assure collisional thermalization of the gas. Such combination of evaporation and sympathetic cooling allows to reach temperatures $T \simeq 0.1T_F$, where T_F is the Fermi temperature at which the gas exhibits quantum degeneracy. Unfortunately, critical temperatures for the BCS transition, T_c , are predicted to be much smaller than T_F . Nowadays, the standard way employs a Feshbach resonance in order to increase the atomic scattering length a_s to larger negative values. Such "resonance superfluidity" should lead to superfluid transition at $T_c \approx 0.1T_F$ [6]. In the most promising scenario, one starts with a molecular condensate formed for $a_s \geq 0$ and modifies a_s towards the negative values [3, 9]. Another way to achieve the BCS regime is to use the cooling scheme that can overcome the Pauli blocking, such as appropriately designed laser cooling [7]. Yet another promising route is to place the Fermi gas in an optical lattice and enter the "high T_c " regime [8].

The temperature of the BCS transition in a two-component Fermi gas depends dramatically on the difference of concentrations of the two components, which presents another experimental obstacle [9]. This problem, however, is not relevant for a polarized Fermi gas with long-range interactions, such as dipole-dipole ones. The possibility of the Cooper pairing has been predicted in Refs. [10] and the critical temperature (including many-body corrections), as well as the order parameter, have been obtained for a homogeneous gas in Ref. [11]. Possible realizations of dipolar gases include ultracold heteronuclear molecular gases [12], atomic gases in a strong DC electric field [13], atomic gases with laser-induced dipoles [14], or with magnetic dipoles [15]. For dipolar moments d of the order of one Debye and densities n of 10^{12}cm^{-3} , T_c should be in the range of 100nK, i.e. experimentally feasible.

Dipole-dipole interaction is not only of long-range, but also anisotropic, i.e. partially attractive and partially repulsive. Thus, the nature of the interaction for trapped gases may be controlled by the geometry of the trap. For a dipolar Bose gas in a cylindrical trap with the axial (radial) frequency $\omega_z(\omega_\rho)$, there exist a critical aspect ratio $\lambda = (\omega_z/\omega_\rho)^{1/2}$, above which the Bose-condensed gas collapses if the atom number is too large [16], and below which the condensate exhibits the roton-maxon instability [17]. The trap geometry is also expected to control the physics of trapped dipolar Fermi gases. So far, however, only partial results were known [18]: analytic corrections to T_c in "loose" traps, and solution of the case of an infinite "slab" with $\omega_\rho = 0$, and ω_z finite. In the latter case there exists a critical frequency above which the superfluid phase does not exist. Very recently, we have reported results for the case of a general trap [19], and predicted the critical dipole strength, below which the BCS pairing transition ceases to exist.

In this paper we present the detailed derivation of the results of Refs. [18] and [19], and study the effect of the trap geometry on the BCS transition in trapped dipolar Fermi gases. We first consider the case of a shallow nearly spherical trap with the trap frequencies ω_z, ω_ρ much smaller than the critical temperature T_c of a spatially homogeneous gas

with the density equal to the maximal density of a trapped gas sample, $\omega_z, \omega_\rho \ll T_c$. In this case, the presence of a confining potential results in a decrease of the critical temperature as compared to the spatially homogeneous gas. In the case of a strong confining potential, where $\omega_z, \omega_\rho \sim T_c$, we calculate the phase diagram in the plane $\Gamma - \lambda^{-1}$, where $\Gamma = 36nd^2/\pi\mu$ is the strength of the dipole-dipole interaction relative to the chemical potential μ . Below the critical value of the interaction, $\Gamma < \Gamma_c$, the BCS transition does not take place. Similarly, for a given dipole interaction strength there is a critical value of λ^{-1} , above which the BCS state appears. We determine the dependence of Γ_c on λ^{-1} , and calculate the order parameter at the criticality. The order parameter exhibits a novel non-monotonic behavior in strongly elongated cylindrical traps.

II. BCS PAIRING IN A DIPOLAR FERMI GAS

We consider a dipolar Fermi gas polarized along the z -direction in a cylindrical trap. The corresponding Hamiltonian reads

$$\begin{aligned} \hat{H} = & \int_{\mathbf{r}} \hat{\psi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2 \nabla^2}{2m} + U_{\text{trap}}(\mathbf{r}) - \mu \right] \hat{\psi}(\mathbf{r}) \\ & + \frac{1}{2} \int_{\mathbf{r}, \mathbf{r}'} \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}^\dagger(\mathbf{r}') V_{\text{dip}}(\mathbf{r} - \mathbf{r}') \hat{\psi}(\mathbf{r}') \hat{\psi}(\mathbf{r}), \end{aligned} \quad (1)$$

where $\hat{\psi}^\dagger(\mathbf{r})$ and $\hat{\psi}(\mathbf{r})$ are the canonical fermionic creation and annihilation operators of particles with the mass m and the dipolar moment d , $U_{\text{trap}}(\mathbf{r}) = m[\omega_\rho^2(x^2 + y^2) + \omega_z^2 z^2]$ the trapping potential, μ the chemical potential, and $V_{\text{dip}}(\mathbf{r}) = (d^2/r^3)(1 - 3z^2/r^2)$.

The BCS pairing is characterized by the order parameter $\Delta(\mathbf{r}_1, \mathbf{r}_2) = V_d(\mathbf{r}_1 - \mathbf{r}_2) \langle \hat{\Psi}(\mathbf{r}_1) \hat{\Psi}(\mathbf{r}_2) \rangle$, which attains nonzero values below the critical temperature T_c . Just below T_c , the order parameter is a nontrivial solution of the BCS gap equation (see. e.g. [1])

$$\Delta(\mathbf{r}_1, \mathbf{r}_2) = -V_{\text{dip}}(\mathbf{r}_1 - \mathbf{r}_2) \int_{\mathbf{r}_3, \mathbf{r}_4} K(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3, \mathbf{r}_4) \Delta(\mathbf{r}_3, \mathbf{r}_4) \quad (2)$$

and is an extremum of the functional

$$F[\Delta] = \int_{\mathbf{r}_1, \mathbf{r}_2} \frac{|\Delta(\mathbf{r}_1, \mathbf{r}_2)|^2}{V_{\text{dip}}(\mathbf{r}_1 - \mathbf{r}_2)} + \int_{\mathbf{r}_1 \dots \mathbf{r}_4} \Delta^*(\mathbf{r}_1, \mathbf{r}_2) K(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3, \mathbf{r}_4) \Delta(\mathbf{r}_3, \mathbf{r}_4). \quad (3)$$

The kernel K in the above expression is

$$K(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_3, \mathbf{r}_4) = T \sum_{\omega_n} G_{\omega_n}(\mathbf{r}_1, \mathbf{r}_3) G_{-\omega_n}(\mathbf{r}_2, \mathbf{r}_4), \quad (4)$$

where

$$G_{\omega_n}(\mathbf{r}, \mathbf{r}') = \sum_{\nu} \frac{\psi_{\nu}(\mathbf{r}) \psi_{\nu}^*(\mathbf{r}')}{i\omega_n - (\varepsilon_{\nu} - \mu)}$$

is the Matsubara Green function of the Fermi gas in the normal phase. In this formula, $\omega_n = \pi T(2n + 1)$, $n = 0, \pm 1, \pm 2, \dots$, $\psi_{\nu}(\mathbf{r})$ and ε_{ν} are the eigenfunctions and the corresponding eigenvalues of the free particle Hamiltonian in the trap, $[-\hbar^2 \nabla^2 / 2m + V_{\text{trap}}(\mathbf{r})] \psi_{\nu}(\mathbf{r}) = \varepsilon_{\nu} \psi_{\nu}(\mathbf{r})$.

The solution of the general problem given by Eqs. (2)–(4), is very difficult even for numerical methods due to the large number of variables, relatively low symmetry of the system, and a long-range character of the interparticle interaction. Remarkably, in the case of a shallow nearly spherical trap, the solution can be found analytically, whereas the general case can be treated by using a variational approach.

III. SHALLOW NEARLY SPHERICAL TRAP

We begin with the case of a weakly deformed spherical trap with the frequencies, which are much less than critical temperature, $\omega_\rho, \omega_z \ll T_c$. In the new variables

$$\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_3)/2, \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_3,$$

$$\mathbf{R}' = (\mathbf{r}_2 + \mathbf{r}_4)/2, \quad \mathbf{r}' = \mathbf{r}_2 - \mathbf{r}_4,$$

the second term in Eq. (3) reads

$$\int_{\mathbf{R}, \mathbf{R}', \mathbf{r}, \mathbf{r}'} \Delta^*(\mathbf{R} + \mathbf{r}/2, \mathbf{R}' + \mathbf{r}'/2) K(\mathbf{R}, \mathbf{R}', \mathbf{r}, \mathbf{r}') \Delta(\mathbf{R} - \mathbf{r}/2, \mathbf{R}' - \mathbf{r}'/2), \quad (5)$$

where

$$K(\mathbf{R}, \mathbf{R}', \mathbf{r}, \mathbf{r}') = T \sum_{\omega_n} G_{\omega_n}(\mathbf{R}, \mathbf{r}) G_{-\omega_n}(\mathbf{R}', \mathbf{r}'). \quad (6)$$

The kernel K depends on variables \mathbf{R} and \mathbf{R}' only due to the presence of the trapping potential, but, as a function of \mathbf{r} and \mathbf{r}' , the kernel decays rapidly for $r, r' > \xi_0$, where $\xi_0 = p_F/mT_c = v_F/T_c$ with $p_F = mv_F = \sqrt{2m\mu}$ being the Fermi momentum, determines the characteristic scale for pairing correlations. Under the condition $\omega_\rho, \omega_z \ll T_c$, the gap $\Delta(\mathbf{r}_1, \mathbf{r}_2) = \Delta(\mathbf{R}_{12}, \mathbf{r}_{12})$ is a slowly varying function (on the scale ξ_0) of $\mathbf{R}_{12} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ (see the end of this Section). At the same time, the Fourier transform of Δ with respect to $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$,

$$\tilde{\Delta}(\mathbf{R}_{12}, \mathbf{p}) = \int_{\mathbf{r}_{12}} \Delta(\mathbf{R}_{12}, \mathbf{r}_{12}) \exp(-i\mathbf{p}\mathbf{r}_{12}), \quad (7)$$

varies on a scale of the order of the Fermi momentum, $p \sim p_F \sim \hbar n^{1/3}$, see Ref. [11]. It is therefore convenient to write Eq. (5) in the form

$$\int_{\mathbf{R}_c} \int_{\mathbf{r}_c, \mathbf{r}, \mathbf{r}'} \int_{\mathbf{P}, \mathbf{q}} \tilde{\Delta}^*(\mathbf{R}_c + \frac{\mathbf{r} + \mathbf{r}'}{4}, \mathbf{P} + \frac{\mathbf{q}}{2}) \exp(i\mathbf{q}\mathbf{r}_c - i\mathbf{P}(\mathbf{r} - \mathbf{r}')) K(\mathbf{R}_c + \frac{\mathbf{r}_c}{2}, \mathbf{R}_c - \frac{\mathbf{r}_c}{2}, \mathbf{r}, \mathbf{r}') \tilde{\Delta}(\mathbf{R}_c - \frac{\mathbf{r} + \mathbf{r}'}{4}, \mathbf{P} - \frac{\mathbf{q}}{2}), \quad (8)$$

where $\mathbf{R}_c = (\mathbf{R} + \mathbf{R}')/2 = (\mathbf{R}_{12} + \mathbf{R}_{34})/2$ and $\mathbf{r}_c = \mathbf{R} - \mathbf{R}'$, and, keeping in mind that the pairing takes place in the central region of the gas sample, where $U_{\text{trap}}(\mathbf{R}) \ll \mu$, we can expand the order parameter in powers of $(\mathbf{r} + \mathbf{r}')/4$.

The leading term of this expansion is

$$\begin{aligned} & \int_{\mathbf{R}_c} \int_{\mathbf{r}_c, \mathbf{r}, \mathbf{r}'} \int_{\mathbf{P}, \mathbf{q}} \tilde{\Delta}^*(\mathbf{R}_c, \mathbf{P} + \frac{\mathbf{q}}{2}) \exp(i\mathbf{q}\mathbf{r}_c - i\mathbf{P}(\mathbf{r} - \mathbf{r}')) T \sum_{\omega_n} G_{\omega_n}(\mathbf{R}_c + \frac{\mathbf{r}_c}{2}, \mathbf{r}) G_{-\omega_n}(\mathbf{R}_c - \frac{\mathbf{r}_c}{2}, \mathbf{r}') \tilde{\Delta}(\mathbf{R}_c, \mathbf{P} - \frac{\mathbf{q}}{2}) \\ &= \int_{\mathbf{R}_c} \int_{\mathbf{P}, \mathbf{q}} \int_{\mathbf{r}_c} \tilde{\Delta}^*(\mathbf{R}_c, \mathbf{P} + \frac{\mathbf{q}}{2}) \exp(i\mathbf{q}\mathbf{r}_c) T \sum_{\omega_n} G_{\omega_n}(\mathbf{R}_c + \frac{\mathbf{r}_c}{2}, \mathbf{P}) G_{-\omega_n}(\mathbf{R}_c - \frac{\mathbf{r}_c}{2}, -\mathbf{P}) \tilde{\Delta}(\mathbf{R}_c, \mathbf{P} - \frac{\mathbf{q}}{2}), \end{aligned} \quad (9)$$

with

$$G_{\omega_n}(\mathbf{R}, \mathbf{P}) = \int_{\mathbf{r}} G_{\omega_n}(\mathbf{R}, \mathbf{r}) \exp(-i\mathbf{P}\mathbf{r}).$$

In the case $\omega_\rho, \omega_z \ll T_c$, the Green function $G_{\omega_n}(\mathbf{R}, \mathbf{P})$ can be approximated as

$$G_{\omega_n}(\mathbf{R}, \mathbf{P}) \approx \frac{1}{i\omega_n - \xi_P + U_{\text{trap}}(\mathbf{R})} = \frac{1}{i\omega_n - (P^2/2m - \mu(\mathbf{R}))} \quad (10)$$

with $\mu(\mathbf{R}) = \mu - U_{\text{trap}}(\mathbf{R})$. With the help of the formula

$$\begin{aligned} T \sum_{\omega_n} \frac{1}{i\omega_n - \xi_P + U_{\text{trap}}(\mathbf{R})} \frac{1}{-i\omega_n - \xi_P + U_{\text{trap}}(\mathbf{R}')} &\approx \left[1 - \frac{U_{\text{trap}}(\mathbf{R}) + U_{\text{trap}}(\mathbf{R}')}{2\mu} \frac{\partial}{\partial \xi_P} \right] \frac{\tanh(\xi_P/2T_c)}{2\xi_P} \\ &= \frac{\tanh(\xi_P/2T_c)}{2\xi_P} - \frac{1}{\mu} (U_{\text{trap}}(\mathbf{R}_c) + U_{\text{trap}}(\mathbf{r}_c/2)) \frac{\partial}{\partial \xi_P} \frac{\tanh(\xi_P/2T_c)}{2\xi_P}, \end{aligned} \quad (11)$$

the integration over \mathbf{r}_c in Eq. (9) gives

$$\int_{\mathbf{r}_c} \exp(i\mathbf{q}\mathbf{r}_c \rho) T \sum_{\omega_n} G_{\omega_n}(\mathbf{R}_c + \mathbf{r}_c/2, \mathbf{P}) G_{-\omega_n}(\mathbf{R}_c - \mathbf{r}_c/2, -\mathbf{P})$$

$$\approx \left(1 - \frac{1}{\mu} U_{\text{trap}}(\mathbf{R}_c) \frac{\partial}{\partial \xi_P}\right) \frac{\tanh(\xi_P/2T_c)}{2\xi_P} \delta(\mathbf{q}) + \frac{1}{\mu} U_{\text{trap}}(\frac{1}{2}\nabla_{\mathbf{q}}) \delta(\mathbf{q}) \frac{\partial}{\partial \xi_P} \frac{\tanh(\xi_P/2T_c)}{2\xi_P}. \quad (12)$$

The term containing $U_{\text{trap}}(\nabla_{\mathbf{q}}/2)$ can be neglected because $\nabla_{\mathbf{q}} \sim 1/p_F$, and, therefore, Eq. (9) can finally be written in the form

$$\int_{\mathbf{R}_c} \int_{\mathbf{P}} \tilde{\Delta}^*(\mathbf{R}_c, \mathbf{P}) \left(1 - \frac{1}{\mu} U_{\text{trap}}(\mathbf{R}_c) \frac{\partial}{\partial \xi_P}\right) \frac{\tanh(\xi_P/2T_c)}{2\xi_P} \tilde{\Delta}(\mathbf{R}_c, \mathbf{P}). \quad (13)$$

This expression corresponds to the local density approximation with $\mu \rightarrow \mu(\mathbf{R}_c) = \mu - U_{\text{trap}}(\mathbf{R}_c)$ expanded in powers of $U_{\text{trap}}(\mathbf{R}_c)/\mu$ up to the first order.

The next term of the expansion of Eq. (8) in powers of $(\mathbf{r} + \mathbf{r}')/4$ is quadratic (the linear in $(\mathbf{r} + \mathbf{r}')/4$ contribution vanishes due to the symmetry of the problem) and has the form

$$\begin{aligned} & \int_{\mathbf{R}_c} \int_{\mathbf{r}_c} \int_{\mathbf{P}, \mathbf{q}} \left(\tilde{\Delta}^*(\mathbf{R}_c, \mathbf{P} + \mathbf{q}/2) \left[\overleftarrow{\nabla}_i \overleftarrow{\nabla}_j + \overrightarrow{\nabla}_i \overrightarrow{\nabla}_j - 2\overleftarrow{\nabla}_i \overrightarrow{\nabla}_j \right] \tilde{\Delta}(\mathbf{R}_c, \mathbf{P} - \mathbf{q}/2) \right) \exp(i\mathbf{q}\mathbf{r}_c) \\ & \times \int_{\mathbf{r}, \mathbf{r}'} \frac{1}{2} \left(\frac{\mathbf{r} + \mathbf{r}'}{4} \right)_i \left(\frac{\mathbf{r} + \mathbf{r}'}{4} \right)_j \exp(-i\mathbf{P}(\mathbf{r} - \mathbf{r}')) K(\mathbf{R}_c + \mathbf{r}_c/2, \mathbf{R}_c - \mathbf{r}_c/2, \mathbf{r}, \mathbf{r}') \end{aligned} \quad (14)$$

where $\overleftarrow{\nabla}_i$ and $\overrightarrow{\nabla}_i$ are the i -th component of the gradient $\nabla_{\mathbf{R}_c}$ acting on the left (on $\tilde{\Delta}^*$) and on the right (on $\tilde{\Delta}$), respectively. After neglecting the q -dependence of $\tilde{\Delta}$, the integrations over \mathbf{r}_c and \mathbf{q} are straightforward and Eq. (14) can be written as

$$\begin{aligned} & \int_{\mathbf{R}_c} \int_{\mathbf{P}} \left(\tilde{\Delta}^*(\mathbf{R}_c, \mathbf{P}) \left[\overleftarrow{\nabla}_i \overleftarrow{\nabla}_j + \overrightarrow{\nabla}_i \overrightarrow{\nabla}_j - 2\overleftarrow{\nabla}_i \overrightarrow{\nabla}_j \right] \tilde{\Delta}(\mathbf{R}_c, \mathbf{P}) \right) \\ & \times \int_{\mathbf{r}, \mathbf{r}'} \frac{1}{2} \left(\frac{\mathbf{r} + \mathbf{r}'}{4} \right)_i \left(\frac{\mathbf{r} + \mathbf{r}'}{4} \right)_j \exp(-i\mathbf{P}(\mathbf{r} - \mathbf{r}')) K(\mathbf{R}_c, \mathbf{R}_c, \mathbf{r}, \mathbf{r}'). \end{aligned} \quad (15)$$

One can show with the help of the explicit form of the Green functions, Eq. (10), that the main contribution from the integrals over \mathbf{r} and \mathbf{r}' is

$$\frac{1}{32} \mathbf{n}_i \mathbf{n}_j \frac{\mu(\mathbf{R}_c)}{mT_c^2} \frac{1}{\cosh^2(\xi_P(\mathbf{R}_c)/2T_c)} \frac{\tanh(\xi_P(\mathbf{R}_c)/2T_c)}{2\xi_P(\mathbf{R}_c)}$$

with \mathbf{n} being the unit vector in the direction of \mathbf{P} , $\mathbf{n} = \mathbf{P}/P$, and $\xi_P(\mathbf{R}_c) = P^2/2m - \mu(\mathbf{R}_c)$. This expression decays exponentially for $|\xi_P(\mathbf{R}_c)| > T_c$, and, therefore, can be approximated by the simpler one in integrals over \mathbf{P} with a slow varying functions of \mathbf{P}

$$\frac{1}{32} \mathbf{n}_i \mathbf{n}_j \frac{\mu(\mathbf{R}_c)}{mT_c^2} \frac{7\zeta(3)}{2\pi^2} \delta(P - p_F(\mathbf{R}_c)),$$

where $\zeta(z)$ is the Riemann zeta-function. With this simplification, Eq. (15) becomes

$$\frac{7\zeta(3)}{64\pi^2} \int_{\mathbf{R}_c} \nu_F(\mu(\mathbf{R}_c)) \frac{\mu(\mathbf{R}_c)}{mT_c^2} \int \frac{d\mathbf{n}}{4\pi} \left(\tilde{\Delta}^*(\mathbf{R}_c, \mathbf{P}) \left[\overleftarrow{\nabla}_i \overleftarrow{\nabla}_j + \overrightarrow{\nabla}_i \overrightarrow{\nabla}_j - 2\overleftarrow{\nabla}_i \overrightarrow{\nabla}_j \right] \tilde{\Delta}(\mathbf{R}_c, \mathbf{P}) \right) \Big|_{\mathbf{P}=\mathbf{n}p_F(\mathbf{R}_c)}, \quad (16)$$

where $\nu_F(\mu(\mathbf{R}_c)) = mp_F(\mathbf{R}_c)/2\pi^2$ is the density of states on the local Fermi surface.

After combining together Eqs. (13) and (16) and performing the variation with respect to $\tilde{\Delta}^*(\mathbf{R}_c, \mathbf{P})$, we obtain the gap equation in the form

$$\begin{aligned} \tilde{\Delta}(\mathbf{R}_c, \mathbf{P}) = & - \int_{\mathbf{P}'} V_{\text{dip}}(\mathbf{P} - \mathbf{P}') \frac{\tanh(\xi_{P'}(\mathbf{R}_c)/2T_c)}{2\xi_{P'}(\mathbf{R}_c)} \tilde{\Delta}(\mathbf{R}_c, \mathbf{P}') \\ & - \frac{7\zeta(3)}{16\pi^2} \nu_F(\mu(\mathbf{R}_c)) \int \frac{d\mathbf{n}'}{4\pi} (\mathbf{n}' \nabla_{\mathbf{R}_c})^2 \tilde{\Delta}(\mathbf{R}_c, \mathbf{P}') \Big|_{\mathbf{P}'=\mathbf{n}'p_F(\mathbf{R}_c)}. \end{aligned} \quad (17)$$

The first line of Eq. (17) coincides with the gap equation in the spatially homogeneous case, see Ref.[11], with \mathbf{R}_c being a parameter. Therefore, following the method developed in Ref.[11], we find that the order parameter on the

local Fermi surface has the form $\tilde{\Delta}(\mathbf{R}_c, \mathbf{P} = \mathbf{n}p_F(\mathbf{R}_c)) = \tilde{\Delta}(\mathbf{R}_c)\varphi_0(\mathbf{n})$, where $\varphi_0(\mathbf{n}) = \sqrt{2}\sin((\pi/2)\cos(\vartheta_{\mathbf{n}}))$ (see Ref. [11] for more details) and the function $\tilde{\Delta}(\mathbf{R}_c)$ obeys the equation

$$\tilde{\Delta}(\mathbf{R}_c) = -\lambda_0(\mathbf{R}_c) \ln \frac{e^C \bar{\omega}(\mathbf{R}_c)}{\pi T_c} \tilde{\Delta}(\mathbf{R}_c) - \frac{7\zeta(3)}{16\pi^2} \lambda_0(0) \int \frac{d\mathbf{n}'}{4\pi} \varphi_0^2(\mathbf{n}') (\mathbf{n}' \nabla_{\mathbf{R}_c})^2 \tilde{\Delta}(\mathbf{R}_c), \quad (18)$$

where $C = 0.5772$ is the Euler constant, $\lambda_0(\mathbf{R}) = (mp_F(\mathbf{R})/2\pi^2)\Gamma_d$, with Γ_d being the dipole-dipole scattering amplitude ($\Gamma_d = -8d^2/\pi$ in the Born approximation). With the account of the expressions for the critical temperature $T_c^{(0)}$ and explicite form of $\varphi_0(\mathbf{n})$ in the spatially homogeneous case, see Ref. [11], the equation (18) for $\tilde{\Delta}(\mathbf{R}_c)$ takes the final form

$$\left\{ -\frac{7\zeta(3)}{48\pi^2} \left(\frac{v_F}{T_c^{(0)}} \right)^2 \sum_{i=1,2,3} f_i \nabla_{\mathbf{R}_{ci}}^2 + \frac{U_{\text{trap}}(\mathbf{R}_c)}{\mu} \left(1 - \frac{1}{2\lambda_0(0)} \right) \right\} \tilde{\Delta}(\mathbf{R}_c) = \ln \frac{T_c^{(0)}}{T_c} \tilde{\Delta}(\mathbf{R}_c), \quad (19)$$

where $f_1 = f_2 = 1 - 3/\pi^2$, $f_3 = 1 + 6/\pi^2$. Note, that in obtaining this equation from Eq. (19), we also expand $\lambda_0(\mathbf{R}_c)$ up to the first order in $U_{\text{trap}}(\mathbf{R}_c)/\mu$, similarly to Eq. (13).

Eq. (19) is equivalent to the Schrödinger equation for a 3D anisotropic harmonic oscillator. As a result, the shift in the critical temperature due to the presence of the trapping potential is given by the lowest eigenvalue and equals

$$\frac{T_c^{(0)} - T_c}{T_c^{(0)}} \approx \ln \frac{T_c^{(0)}}{T_c} = \frac{1}{T_c^{(0)}} \sqrt{\frac{7\zeta(3)}{48\pi^2} \left(1 + \frac{1}{2|\lambda_0|} \right)} \left[2\omega_\rho \sqrt{1 - \frac{3}{\pi^2}} + \omega_z \sqrt{1 + \frac{6}{\pi^2}} \right]. \quad (20)$$

In the considered case $\omega_i/T_c \ll 1$, the critical temperature in the trap is only slightly lower than that in the spatially homogeneous case.

Just below T_c , the order parameter has the gaussian form

$$\tilde{\Delta}(\mathbf{R}_c) \propto \exp\left(-\sum_{i=1,2,3} R_{ci}^2/2l_{\Delta i}^2\right),$$

where $l_{\Delta i} = (v_F/\omega_i)\sqrt{\omega_i/T_c^{(0)}} [7\zeta(3)f_i/48\pi^2(1 + 1/2|\lambda_0|)]^{1/4}$ is the characteristic size in the i -th direction. One can see, that $l_{\Delta i} \ll R_{TF}^{(i)}$, where $R_{TF}^{(i)} = v_F/\omega_i$ is the Thomas-Fermi radius of the gas sample in the i -th direction. This justifies our assumption that pairing takes place only in the central part of the sample. On the other hand, we have $l_{\Delta i} \gg \xi_0$ and, therefore, the gradient expansion of the order parameter in powers of $\mathbf{r} + \mathbf{r}'$ is legitimate.

For a given number of particle $N = \mu^3/3\omega_z\omega_\rho^2$ in the trap with aspect ratio $\lambda = \sqrt{\omega_z/\omega_\rho}$, we have $\omega_z = \omega\lambda^{4/3}$ and $\omega_\rho = \omega\lambda^{-2/3}$, where $\omega = (\omega_z\omega_\rho^2)^{1/3}$, and Eq. (20) becomes

$$\frac{T - T_c^{(0)}}{T_c^{(0)}} = -\frac{\omega}{T_c^{(0)}} \sqrt{\left(1 + \frac{1}{2|\lambda_0|}\right)} F(\lambda), \quad (21)$$

where $F(\lambda) = \sqrt{7\zeta(3)/48\pi^2} [2\sqrt{1 - 3/\pi^2}\lambda^{-2/3} + \sqrt{1 + 6/\pi^2}\lambda^{4/3}]$. The plot of the function $F(\lambda)$ is presented on Fig. 1.

We see that there exists the optimal value of the trap aspect ration $\lambda^* = 0.81$, which corresponds to the highest transition temperature in the trap. The existence of the optimal value for the trap aspect ratio is a result of the competition between the anisotropic character of the dipole-dipole interparticles interaction and the confinement of the gas sample in the radial direction. The former becomes predominantly attractive for smaller values of λ (cylindrical traps) and, therefore, favours the BCS pairing. The latter, on the contrary, acts on the pairing destructively due to the size effect and, therefore, less pronounced for larger values of λ .

IV. CRITICAL ASPECT RATIO

We have seen in the previous section, that the trap geometry has a more pronounced influence on the BCS pairing in a dipolar Fermi gas, as compared to a two-component Fermi gas with a short-range interaction. This is due to the fact that the states, which form Cooper pairs in a trapped dipolar Fermi gas, have different quantum numbers n_z . Therefore, their energies are different, at least by the amount of ω_z . When this difference becomes of the order of T_c ,

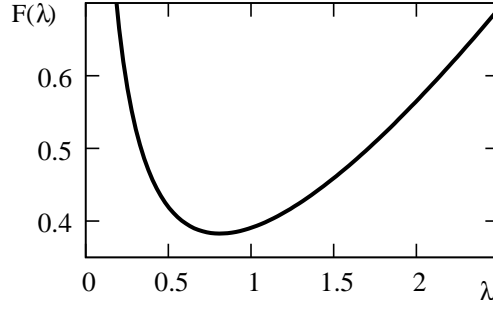


FIG. 1: The function $F(\lambda)$ versus the trap aspect ratio λ .

the pairing is obviously impossible. As a result, the superfluid transition in a trapped dipolar Fermi gas can take place only in traps with $\omega_z < \omega_{zc}$, where the critical frequency ω_{zc} is found to be $\omega_{zc} = 1.8T_c$ [18]. For $\omega_z < \omega_{zc}$, as can be seen from Eq.(21), the confinement in the radial direction decreases the critical temperature as well. Therefore, one would expect the existence of the critical aspect ratio λ_c such that the pairing is possible only in traps with $\lambda < \lambda_c$ [19].

In this Section we study the BCS pairing in the case of a cylindrical trap where the trap frequencies can be of the order of the critical temperature in the spatially homogeneous gas, $\omega_\rho, \omega_z \sim T_c^{(0)}$, but still much less than the chemical potential, $\omega_\rho, \omega_z \ll \mu$. In this case, the BCS gap equation (2) can hardly be tractable even numerical without additional simplifications.

As it was shown in Ref. [11], the BCS pairing is dominated by the p -wave scattering with zero projection of the angular momentum on the z -axis, $l_z = 0$, in which the interaction is attractive. Contributions of higher angular momenta, although present due to the long-range character of the dipole-dipole interaction, are numerically small (see also Ref. [10]). In the p -wave channels with $l_z = \pm 1$ the interaction is repulsive and, therefore, leads only to renormalizations of a Fermi-liquid type, and will be neglected here. Therefore, for the considered pairing problem we can model the dipole-dipole interaction by the following (off-shell) scattering amplitude

$$\Gamma_1(\mathbf{p}, \mathbf{p}', E) = p_z p'_z \tilde{\gamma}_1(E), \quad (22)$$

where \mathbf{p} is the incoming momentum, \mathbf{p}' the outgoing one, and $\tilde{\gamma}_1(E)$ some function of the energy E . The amplitude Γ_1 describes anisotropic scattering only in the p -wave channel with the projection of the angular momentum $l_z = 0$. The function $\tilde{\gamma}_1(E)$ obeys the equation

$$\tilde{\gamma}_1(E) - \tilde{\gamma}_1(E') = \int^\Lambda \frac{d\mathbf{p}}{(2\pi)^3} \tilde{\gamma}_1(E) \left(\frac{p_z^2}{p^2 - E + i0} - \frac{p_z^2}{p'^2 - E' + i0} \right) \tilde{\gamma}_1(E'), \quad (23)$$

that follows from the Lipmann-Schwinger equation for the off-shell scattering amplitude [20]; $\tilde{\gamma}_1(E)$ is assumed to be negative in order to guarantee the BCS pairing. The cut-off parameter Λ ensures the convergence of the integral and, in fact, can be expressed in terms of the observable scattering data corresponding to on-shell scattering amplitude with $p = p'$ and $E = p^2/m$. It follows from Eq. (23) that $\tilde{\gamma}_1(E)$ is inversely proportional to E , $\tilde{\gamma}_1(E) = \gamma_1 (2mE)^{-1}$, with some coefficient γ_1 . Therefore, the on-shell amplitude is energy independent, as it should be for low-energy scattering on the dipole-dipole potential (see Ref. [21]).

The coefficient γ_1 determines the value of the critical temperature T_c of the BCS transition in a spatially homogeneous gas and can be expressed through the dipole moment d using the results of Ref. [11]. In a homogeneous gas, the order parameter has the form $\Delta(\mathbf{p}) = p_z \Delta_0$ with some constant Δ_0 , and the linearized gap equation implies

$$\frac{1}{\tilde{\gamma}_1(\mu)} = - \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{p_z^2}{2\xi_p} \left[\tanh \frac{\xi_p}{2T_c} - 1 \right], \quad (24)$$

where $\xi_p = p^2/2m - \mu$, and the bare interaction is renormalized in terms of the scattering amplitude with $\tilde{\gamma}_1(\mu) = \gamma_1/p_F^2$ at the Fermi energy $\varepsilon_F = \mu = p_F^2/2m$ along the lines of Ref. [22] (p_F is the Fermi momentum). After integrating over p , we obtain the equation on T_c :

$$1 = \frac{1}{3} |\gamma_1| \nu_F \left[\ln \frac{2\mu}{T_c} - \frac{8}{3} - \ln \frac{\pi}{4} + C \right], \quad (25)$$

where $\nu_F = mp_F/2\pi^2$ is the density of states at the Fermi energy. After comparing the solution of Eq. (25) with the result of Ref. [11] for T_c , we find $\gamma_1 = -24d^2/\pi$.

In the ordinary space, the scattering amplitude Γ_1 is

$$\Gamma_1(\mathbf{r}, \mathbf{r}', E) = \partial_z \delta(\mathbf{r}) \partial_{z'} \delta(\mathbf{r}') \tilde{\gamma}_1(E), \quad (26)$$

where \mathbf{r} and \mathbf{r}' are the relative distances between the two incoming and the two outgoing particles, respectively. Therefore, the order parameter in the trapped gas, $\Delta(\mathbf{r}_1, \mathbf{r}_2) \sim \langle \psi(\mathbf{r}_1) \psi(\mathbf{r}_2) \rangle$, has the form

$$\Delta(\mathbf{r}_1, \mathbf{r}_2) = \partial_z \delta(\mathbf{r}) \Delta_0(\mathbf{R}), \quad (27)$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, and the corresponding equation for the critical temperature is

$$\begin{aligned} \frac{\Delta_0(\mathbf{R})}{\tilde{\gamma}_1(\mu)} = & - \int_{\mathbf{R}'} \left[\sum_{\mathbf{n}_1, \mathbf{n}_2} M_{\mathbf{n}_1, \mathbf{n}_2}(\mathbf{R}) M_{\mathbf{n}_1, \mathbf{n}_2}(\mathbf{R}') \frac{\tanh(\xi_1/2T) + \tanh(\xi_2/2T)}{2(\xi_1 + \xi_2)} \right. \\ & \left. - \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{p_z^2}{2\xi_p + q^2/4m} \exp(i\mathbf{q}(\mathbf{R} - \mathbf{R}')) \right] \Delta_0(\mathbf{R}'). \end{aligned} \quad (28)$$

Here $\xi_i = \xi(\mathbf{n}_i)$, $\mathbf{n} = (n_x, n_y, n_z)$ are the harmonic oscillator quantum numbers, $\xi(\mathbf{n}) = \hbar[\omega_z(n_z + 1/2) + \omega_\rho(n_x + n_y + 1)] - \mu$, and the function $M_{\mathbf{n}_1, \mathbf{n}_2}(\mathbf{R})$ is defined as

$$M_{\mathbf{n}_1, \mathbf{n}_2}(\mathbf{R}) = M_{n_{1z}n_{2z}}^{(z)}(z) M_{n_{1x}n_{2x}}^{(\rho)}(x) M_{n_{1y}n_{2y}}^{(\rho)}(y) \quad (29)$$

with

$$M_{n_1 n_2}^{(z)}(z) = \frac{1}{2} [\varphi_{n_1}(z) \partial_z \varphi_{n_2}(z) - \varphi_{n_2}(z) \partial_z \varphi_{n_1}(z)], \quad M_{n_1 n_2}^{(\rho)}(x) = \varphi_{n_1}(x) \varphi_{n_2}(y), \quad (30)$$

$\varphi_n(z)$ being the harmonic oscillator wave functions.

The gap equation (28) is still hardly tractable numerically and, hence, we employ additional approximations. We assume a large number of particles such that the chemical potential μ is much larger than the trap frequencies, $\mu \gg \omega_z, \omega_\rho$. Therefore, while calculating the functions $M_{\mathbf{n}_1, \mathbf{n}_2}(\mathbf{R})$, we can use the WKB approximation for the wave functions φ_n of the most important for the BCS pairing states with energies near the Fermi energy $\varepsilon_F = \mu$. Another simplification is due to the fact that the BCS order parameter $\Delta_0(\mathbf{R})$ varies slowly on an interparticle distance scale $n^{-1/3} \sim \hbar/p_F$, where $p_F = \sqrt{2m\mu}$ is now the Fermi momentum in the center of the trap in the Thomas-Fermi approximation. As a result, the pairing correlations are pronounced only between states that are close in energy. This allows to neglect $q^2/4m$ in the denominator of the second term in Eq.(28) together with rapidly oscillating terms in the functions $M_{\mathbf{n}_1, \mathbf{n}_2}(\mathbf{R})$. We then obtain (see Appendix)

$$M_{n_1 n_2}^{(z)}(z) \equiv M_{Nn}^{(z)}(z) \approx (-1)^n \frac{m\omega_z}{\pi} \sqrt{1 - (z/l_{zN})^2} U_{n-1}(z/l_{zN}) \quad (31)$$

and

$$M_{n_1 n_2}^{(\rho)}(x) \equiv M_{Nn}^{(\rho)}(x) \approx (-1)^n \frac{1}{\pi l_{\rho N}} \frac{1}{\sqrt{1 - (x/l_{\rho N})^2}} T_n(x/l_{\rho N}), \quad (32)$$

where $n \equiv |n_1 - n_2| \ll N \equiv (n_1 + n_2)/2$, $l_{iN} = \sqrt{2N\hbar/m\omega_i} = l_{0i}\sqrt{2N}$, whereas $U_n(z) = \sin((n+1)\arccos z)/\sin(\arccos z)$ and $T_n(x) = \cos(n\arccos x)$ are the Chebyshev polynomials. The functions $M_{n_1 n_2}^{(z)}(z)$ and $M_{n_1 n_2}^{(\rho)}(x)$ fulfill the following completeness relations

$$\sum_{n \geq 1} M_{Nn}^{(z)}(z) M_{Nn}^{(z)}(z') = \frac{(n\omega_z)^2}{2\pi} \sqrt{l_{zN}^2 - z^2} \delta(z - z') \quad (33)$$

$$\sum_{n \geq 0} \delta_n M_{Nn}^{(\rho)}(x) M_{Nn}^{(\rho)}(x') = \frac{1}{\pi} \frac{1}{\sqrt{l_{\rho N}^2 - x^2}} \delta(x - x'), \quad (34)$$

with $\delta_0 = 1$ and $\delta_{n>0} = 2$, which follow from the completeness of the Chebyshev polynomials.

It is convenient to rewrite Eq. (28) in the following way

$$\begin{aligned}
-\frac{\Delta_0(\mathbf{R})}{\tilde{\gamma}_1(\mu)} &\approx \int d\mathbf{R}' \left[\sum_{\mathbf{n}_1, \mathbf{n}_2} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}) \left\{ \frac{\tanh(\xi_1/2T) + \tanh(\xi_2/2T)}{2(\xi_1 + \xi_2)} - \frac{\tanh((\xi_1 + \xi_2)4T)}{\xi_1 + \xi_2} \right\} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}') \right. \\
&\quad + \sum_{\mathbf{n}_1, \mathbf{n}_2} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}) \left\{ \frac{\tanh((\xi_1 + \xi_2)4T)}{2(\xi_1 + \xi_2)} - \frac{1}{\xi_1 + \xi_2} \right\} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}') \\
&\quad \left. \sum_{\mathbf{n}_1, \mathbf{n}_2} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}) \frac{1}{\xi_1 + \xi_2} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}') - \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{p_z^2}{2(p^2/2m - \mu)} \exp(i\mathbf{q}(\mathbf{R} - \mathbf{R}')) \right] \Delta_0(\mathbf{R}') \\
&\equiv \int d\mathbf{R}' [K_1(\mathbf{R}, \mathbf{R}') + K_2(\mathbf{R}, \mathbf{R}') + K_3(\mathbf{R}, \mathbf{R}')] \Delta_0(\mathbf{R}'), \tag{35}
\end{aligned}$$

because the sum $\xi_1 + \xi_2$ does not depend on $n_1 - n_2$ and, therefore, with the help of formulae (33) and (34), the summation over n can easily be performed in kernels K_2 and K_3 . On the other hand, the kernel K_1 is determined entirely by the states near the chemical potential μ .

The calculation of the sums in the kernel $K_3(\mathbf{R}, \mathbf{R}')$ gives

$$\begin{aligned}
&\sum_{\mathbf{n}_1, \mathbf{n}_2} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}) \frac{1}{\xi_1 + \xi_2} M_{\mathbf{n}_1 \mathbf{n}_2}(\mathbf{R}') = \sum_{\mathbf{N}} \sum_{\mathbf{n}_1 - \mathbf{n}_2} M_{\mathbf{N} \mathbf{n}}(\mathbf{R}) \frac{1}{2\xi(\mathbf{N})} M_{\mathbf{N} \mathbf{n}}(\mathbf{R}') \\
&= \delta(\mathbf{R} - \mathbf{R}') \sum_{N_i \geq R_i^2/2l_{0i}^2} \frac{1}{2\xi(\mathbf{N})} \frac{(n\omega_z)^2}{\pi^3} \sqrt{\frac{l_{zN}^2 - z^2}{(l_{\rho N}^2 - x^2)(l_{\rho N}^2 - y^2)}} = \delta(\mathbf{R} - \mathbf{R}') \sum_{N_i \geq 0} \frac{1}{2\xi(\mathbf{N})} \frac{(n\omega_z)^2}{\pi^3} \frac{l_{zN}}{l_{\rho N} l_{\rho N}} \\
&= \delta(\mathbf{R} - \mathbf{R}') \int_0 \prod_i \frac{dp_i}{\pi} \frac{p_z^2}{2(p^2/2m - \mu(\mathbf{R}))},
\end{aligned}$$

where $\mu(\mathbf{R}) = \mu - \sum_i m\omega_i^2 R_i^2/2$, and we have replaced the discrete sums over N_i with the integrals over continuous variables $p_i = \sqrt{2m\omega_i N_i}$. As a result, the kernel $K_3(\mathbf{R}, \mathbf{R}')$ is

$$K_3(\mathbf{R}, \mathbf{R}') \approx \int \frac{d\mathbf{p}}{(2\pi)^3} \left[\frac{p_z^2}{2(p^2/2m - \mu(\mathbf{R}))} - \frac{p_z^2}{2(p^2/2m - \mu)} \right] \delta(\mathbf{R} - \mathbf{R}'). \tag{36}$$

Following Eq.(23), we see that the kernel K_3 results in the replacement $\mu \rightarrow \mu(\mathbf{R})$ in the scattering amplitude $\tilde{\gamma}_1(\mu)$ on the left-hand-side of the gap equation (35).

We are interested in the critical value λ_c of the aspect ratio, below which the BCS pairing does not take place for a given strength of the dipole interaction. Therefore, this value corresponds to vanishing critical temperature. We therefore calculate the kernels K_1 and K_2 in the limit $T \ll \omega_i$. The summation over $\xi(\mathbf{N}) = (\xi_1) + \xi_1)/2$ in the kernel K_2 is then within the limits $-\mu \leq \xi(\mathbf{N}) \leq -\omega_z/2$, where $-\omega_z/2$ is the upper limit due to the fact that the function $M_{n_1 n_2}^{(z)}$ is nonzero only when $|n_{1z} - n_{2z}| \geq 1$. These sums can again be replaced by integrals with the following result

$$K_2(\mathbf{R}, \mathbf{R}') \approx \frac{1}{3} p_F^2(\mathbf{R}) \nu_F(\mathbf{R}) \left\{ \ln \frac{2\mu(\mathbf{R})}{\omega_z} - \frac{2}{3} (4 - 2 \ln 2) \right\} \delta(\mathbf{R} - \mathbf{R}'), \tag{37}$$

where $p_F(\mathbf{R}) = \sqrt{2m\mu(\mathbf{R})}$ is the local Fermi momentum in the Thomas-Fermi approximation.

In order to calculate the kernel K_1 in the limit $T \ll \omega_i$ we note, that nonzero contributions to the sums over \mathbf{n}_1 and \mathbf{n}_2 originate only from the region $\omega_z/2 \leq |\xi(\mathbf{N})| \leq \varepsilon(\mathbf{n})/2$, where $n_i = |n_{1i} - n_{2i}|$ and $\varepsilon(\mathbf{n}) = \hbar \sum_i \omega_i n_{1i}$. As a result, the kernel $K_1(\mathbf{R}, \mathbf{R}')$ can be written as

$$\begin{aligned}
K_1(\mathbf{R}, \mathbf{R}') &= - \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \sum_{\omega_z/2 \leq \xi(\mathbf{N}) \leq \varepsilon(\mathbf{n})/2} \frac{1}{\xi(\mathbf{N})} M_{\mathbf{N} \mathbf{n}}(\mathbf{R}) M_{\mathbf{N} \mathbf{n}}(\mathbf{R}') \\
&= - \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \int_{\omega_z/2}^{\varepsilon(\mathbf{n})/2} \frac{d\xi}{\xi} \sum_{\mathbf{N}} \delta(\xi - \xi(\mathbf{N})) M_{\mathbf{N} \mathbf{n}}(\mathbf{R}) M_{\mathbf{N} \mathbf{n}}(\mathbf{R}').
\end{aligned}$$

For a generic cylindrical trap, $f(\xi, \mathbf{n}) = \sum_{\mathbf{N}} \delta(\xi - \xi(\mathbf{N})) M_{\mathbf{Nn}}(\mathbf{R}) M_{\mathbf{Nn}}(\mathbf{R}')$ is a smooth function of ζ (this is equivalent to neglecting the so-called shell effect). At the same time, only small \mathbf{n} with $\varepsilon(\mathbf{n}) \ll \mu$ contributes to the kernel K_1 . We therefore can replace $f(\xi, \mathbf{n})$ with $f(0, \mathbf{n})$. After replacing the sums over N_i , $i = x, y, z$, with the integrals over $\alpha = \omega_\rho N_x / \mu$, $\beta = \omega_\rho N_y / \mu$, and $\zeta = \omega_z N_z / \mu$, we obtain

$$\begin{aligned} K_1(\mathbf{R}, \mathbf{R}') &\approx - \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \ln \frac{\varepsilon(\mathbf{n})}{\omega_z} \sum_{\mathbf{N}} \delta(\xi(\mathbf{N})) M_{\mathbf{Nn}}(\mathbf{R}) M_{\mathbf{Nn}}(\mathbf{R}') \\ &\approx - \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \frac{\mu^2}{\omega_\rho^2 \omega_z} \ln \frac{\varepsilon(\mathbf{n})}{\omega_z} \int_0^1 d\alpha d\beta d\zeta \delta(1 - \alpha - \beta - \zeta) M_{\mathbf{Nn}}(\mathbf{R}) M_{\mathbf{Nn}}(\mathbf{R}'), \end{aligned} \quad (38)$$

where $N_x = \alpha \mu / \omega_\rho$, $N_y = \beta \mu / \omega_\rho$, and $N_z = \zeta \mu / \omega_z$.

We write the order parameter in the form $\Delta_0(\mathbf{R}) = \Delta_0(z R_{TF}^{(z)}, x R_{TF}^{(\rho)}, y R_{TF}^{(\rho)}) = \Delta_0(\mathbf{r})$, where $R_{TF}^{(i)} = p_F / m \omega_i$ is the Thomas-Fermi radius of the gas cloud in the i -direction and $|x|, |y|, |z| \leq 1$ are dimensionless variables. After combining together Eqs.(35),(38),(37), and (36), the gap equation in limit $T \ll \omega_i$ reads

$$\frac{3}{\Gamma} (1 - r^2) \Delta_0(\mathbf{r}) = (1 - r^2)^{3/2} \left[\ln \frac{2\mu(\mathbf{r})}{\omega_z} - \frac{2}{3} (4 - \ln 4) \right] \Delta_0(\mathbf{r}) - \frac{3\pi^2}{2} \int_{\mathbf{r}'} K(\mathbf{r}, \mathbf{r}') \Delta_0(\mathbf{r}'), \quad (39)$$

where $\Gamma = |\gamma_1| \nu_F$, $\mu(\mathbf{r}) = \mu - V_{\text{trap}}(\mathbf{r})$, and

$$\begin{aligned} K(\mathbf{r}, \mathbf{r}') &= \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \ln \left[n_z + \frac{\omega_\rho}{\omega_z} (n_x + n_y) \right] \\ &\times \int_{M_z}^1 d\zeta \int_{M_x}^1 d\alpha \int_{M_y}^1 d\beta \delta(1 - \zeta - \alpha - \beta) \\ &\times \frac{4}{\pi^2} \frac{\sqrt{(\zeta - z^2)(\zeta - z'^2)}}{\zeta} U_{n_z-1}\left(\frac{z}{\sqrt{\zeta}}\right) U_{n_z-1}\left(\frac{z'}{\sqrt{\zeta}}\right) \\ &\times \frac{4}{\pi^2} \frac{1}{\sqrt{(\alpha - x^2)(\alpha - x'^2)}} T_{n_x}\left(\frac{x}{\sqrt{\alpha}}\right) T_{n_x}\left(\frac{x'}{\sqrt{\alpha}}\right) \\ &\times \frac{4}{\pi^2} \frac{1}{\sqrt{(\beta - y^2)(\beta - y'^2)}} T_{n_y}\left(\frac{y}{\sqrt{\beta}}\right) T_{n_y}\left(\frac{y'}{\sqrt{\beta}}\right), \end{aligned} \quad (40)$$

with $M_s = \max(s^2, s'^2)$ for $s = x, y, z$.

Before solving the above equation numerically, let us analyse the behaviour of its solutions near the edge of the gas sample, $r \rightarrow 1$. In this region $\max(x^2, x'^2) = x^2$, $\max(y^2, y'^2) = y^2$, $\max(z^2, z'^2) = z^2$, and it is convenient to introduce the new variables $a = \alpha - x^2$, $b = \beta - y^2$, and $c = \zeta - z^2$. The delta function in Eq. (40) then reads $\delta(1 - r^2 - a - b - c)$ and, therefore, only small $a, b, c \leq 1 - r^2 \rightarrow 0$ contribute to the integral. This allows us to write the integral in the last term in Eq. (39) in the form

$$\begin{aligned} \int d\mathbf{r}' K(\mathbf{r}, \mathbf{r}') \Delta_0(\mathbf{r}') &\approx \left(\frac{4}{\pi^2} \right)^3 \frac{2}{3\pi^2} \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \ln \left[n_z + \frac{\omega_\rho}{\omega_z} (n_x + n_y) \right] \int_0^{1-r^2} da db dc \sqrt{\frac{c}{ab}} \delta(1 - r^2 - a - b - c) \\ &\times \int d\mathbf{r}' \sqrt{\frac{1 - z'^2}{(1 - x'^2)(1 - y'^2)}} U_{n_z-1}(1) U_{n_z-1}(z') T_{n_x}(1) T_{n_x}(x') T_{n_y}(1) T_{n_y}(y') \Delta_0(xx', yy', zz') \\ &\approx (1 - r^2)^{3/2} \frac{512}{3\pi^5} \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \ln \left[n_z + \frac{\omega_\rho}{\omega_z} (n_x + n_y) \right] U_{n_z-1}(1) T_{n_x}(1) T_{n_y}(1) \\ &\times \int d\mathbf{r}' \sqrt{\frac{1 - z'^2}{(1 - x'^2)(1 - y'^2)}} U_{n_z-1}(z') T_{n_x}(x') T_{n_y}(y') \Delta_0(xx', yy', zz'), \end{aligned} \quad (41)$$

and simple analysis of Eq. (39) gives the asymptotics $\Delta_0(\mathbf{r}) \sim (1 - r^2)^{1/2}$ for $r \rightarrow 1$. Eq. (39) in the limit $r \rightarrow 0$ and Eq. (41) can be thus used as a consistency test for numerical solutions of Eq. (39).

Our numerical approach to Eq.(39) is based on the observation that it can be considered as the equation on extrema of the quadratic form

$$F[\Delta_0] = \frac{1}{2} \int_{\mathbf{r}, \mathbf{r}'} \Delta_0(\mathbf{r}) [L(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') - K(\mathbf{r}, \mathbf{r}')] \Delta_0(\mathbf{r}')$$

with $L(\mathbf{r}) = 3(1 - r^2)/\Gamma - (1 - r^2)^{3/2} \{ \ln([2\mu(\mathbf{r})/\omega_z] - 2(4 - \ln 4)/3) \}$. Taking into account the asymptotics of the solutions of Eq.(39), we will use the ansatz

$$\Delta_0(\mathbf{r}) = (1 - r^2)^{1/2} \sum_{m_z, m_\rho \geq 0}^{M_z, M_\rho} c_{m_z m_\rho} U_{m_z}(z^2) T_{m_\rho}(x^2 + y^2) \quad (42)$$

and write the functional $F[\Delta_0]$ as a quadratic form of unknown coefficients $c_{m_z m_\rho}$

$$F[c] = \sum_{n_z, n_\rho, m_z, m_\rho \geq 0} M_{m_z, m_\rho, n_z, n_\rho} c_{m_z m_\rho} c_{n_z n_\rho}$$

with

$$\begin{aligned} M_{m_z, m_\rho, n_z, n_\rho} = & \frac{1}{2} \int d\mathbf{r} \left\{ \frac{3(1 - r^2)}{\Gamma} - (1 - r^2)^{3/2} \left[\ln \frac{2\mu}{\omega_z} (1 - r^2) - \frac{2}{3} (4 - 2 \ln 2) \right] \right\} \\ & U_{m_z}(z^2) T_{m_\rho}(x^2 + y^2) U_{n_z}(z^2) T_{n_\rho}(x^2 + y^2) \\ & - \frac{3\pi^2}{2} \sum_{n_z > 0; n_x, n_y \geq 0} \delta_{n_x} \delta_{n_y} \ln[n_z + \frac{\omega_\rho}{\omega_z} (n_x + n_y)] \\ & \int_0^1 d\alpha d\beta d\zeta \delta(1 - \alpha - \beta - \zeta) \zeta V_{\mathbf{n}}^{m_z, m_\rho}(\alpha, \beta, \zeta) V_{\mathbf{n}}^{n_z, n_\rho}(\alpha, \beta, \zeta), \end{aligned} \quad (43)$$

where the functions V are defined as:

$$V_{\mathbf{n}}^{n_z, n_\rho}(\alpha, \beta, \zeta) = \int_{-1}^1 \frac{dx}{\pi} \frac{T_{n_x}(x)}{\sqrt{1 - x^2}} \int_{-1}^1 \frac{dy}{\pi} \frac{T_{n_y}(y)}{\sqrt{1 - y^2}} \int_{-1}^1 \frac{dz}{\pi} \sqrt{1 - z^2} U_{n_z-1}(z) U_{n_z}(\zeta z^2) T_{n_\rho}(\alpha x^2 + \beta y^2) \quad (44)$$

An extremum of this form at T_c corresponds to the eigenvector of the matrix $M_{m_z, m_\rho, n_z, n_\rho}$ with a zero eigenvalue.

The condition that the matrix M has a zero eigenvalue imposes a constraint on the interaction parameter Γ and the trap frequencies ω_i and, therefore, allows to determine the critical aspect ratio. Indeed, the parameter Γ can be written as $\Gamma = 36n(0)d^2/\pi\mu$, where $n(0) = (2m\mu)^{3/2}/6\pi^2\hbar^3$ is the gas density in the center of the trap, and, hence, for a given dipole moment d , Γ depends only on the chemical potential μ . On the other hand, the chemical potential μ and the total number of particles in the trap N are also related, $3N = \mu^3/\omega_z\omega_\rho^2$. As a result, for a given Γ and N , the product of the trap frequencies $\omega_z\omega_\rho^2$ is completely determined, and the only free parameter is the trap aspect ratio $\lambda = (\omega_z/\omega_\rho)^{1/2}$. We may thus determine its critical value λ_c from the condition that the lowest eigenvalue is zero.

The calculation of the matrix elements $M_{m_z, m_\rho, n_z, n_\rho}$ is naturally divided into two parts (see Eq. (39)). The contribution with the local kernel $L(\mathbf{r})$ is a three-dimensional integral that can easily be computed using, for instance, the Monte Carlo integration routine, such as the VEGAS algorithm from the GSL library [23]. The part with the non-local kernel $K(\mathbf{r}, \mathbf{r}')$ is a triple sum, which elements are eight-dimensional integrals. The straightforward approach based on the same numerical method fails in this case because it is too time consuming. To overcome this problem, we calculate the functions $V_{\mathbf{n}}^{n_z, n_\rho}(\alpha, \beta, \zeta)$ in the following way. We integrate numerically over \mathbf{r}' for fixed α, β and n 's (the value of ζ is then fixed by the δ -function) and use these data for a two-dimensional spline interpolation of the integrand for the last integrations over α and β . In this way, the time required to compute the matrix elements $M_{m_z, m_\rho, n_z, n_\rho}$ reduces to about seventy two hours on a standard workstation. This procedure gives convergent results with respect to the highest powers of the polynomials M_ζ and M_ρ in our ansatz, Eq. (42). We also checked that our solutions do not depend on the number of shots in the Monte Carlo algorithm and on the number of points chosen for interpolation. They are also a proper asymptotic behavior for $r \rightarrow 0$.

The results of our calculations are presented in 2 figures. Fig.2 shows the desired relation between the interaction strength Γ and the inverse critical aspect ratio λ_c^{-1} . The three curves correspond to three different numbers of

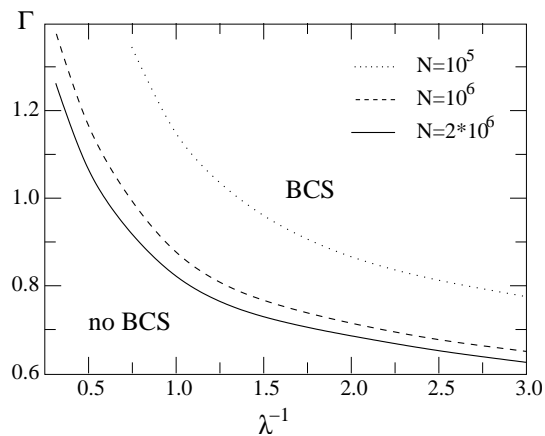


FIG. 2: The critical lines Γ_c versus the inverse aspect ratio λ^{-1} for different numbers of particles. The BCS pairing takes place above the depicted curves.

particles. As it could be expected, for larger number of particles, the critical aspect ratio λ_c is smaller because the interaction is stronger. For a pancake trap, $\lambda^{-1} < 1$, the interaction is predominantly repulsive, and higher values of Γ for a fixed λ are required to achieve the BCS transition. On the other hand, for a cigar trap, $\lambda^{-1} > 1$, the interaction is predominantly attractive and the BCS transition takes place at smaller values of Γ . The existence of the critical interaction strength (for a given value of λ) is a result of the discreteness of the spectrum in the trap and of the specific structure of the order parameter ($\sim p_z$). The latter allows pairing only between particles in the states, where quantum numbers n_z differ by an odd number (intershell pairing). Therefore, the pairing correlations have to be strong enough to overcome the corresponding energy difference.

Fig. 3 shows the order parameter $\Delta_0(\mathbf{r})$ for the cigar trap with $\lambda^{-1} = 2.2$. The order parameter is a non-monotonic function of the distance from the trap center, in contrast to the case of the BCS order parameter in a two component Fermi gas with short range interactions [24]. This effect persists, although being less pronounced, for the case of a pancake geometry. In the axial direction, the order parameter $\Delta(z, \rho = 0)$ develops a minimum at $\rho < 1$, whereas in the radial direction $\Delta(z = 0, \rho)$ becomes negative in the outer part of the cloud. This completely new behavior, originating from the anisotropy of the interparticle interaction, can have profound consequences for the form and spectrum of the elementary excitations.

V. CONCLUSIONS

We have presented a detailed theory and analyzed the influence of a trapping potential on the BCS pairing in a dipolar single-component Fermi gas. We have determined the phase diagram for trapped dipolar Fermi gases in the $\Gamma - \lambda^{-1}$ plane, where Γ measures the dipole strength and λ is the trap aspect ratio. The BCS transition at finite temperature T is possible iff $\Gamma > \Gamma_c(\lambda)$. We have calculated the critical line $\Gamma_c(\lambda)$, and the order parameter at criticality.

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VII. APPENDIX

Here we present the derivation of the WKB-expressions for the functions $M_{n_1 n_2}^{(\rho)}(x)$ and $M_{n_1 n_2}^{(z)}(z)$, Eqs. (32) and (31). Our starting point is the WKB-eigenfunctions $\varphi_n(x)$ of a one-dimensional harmonic oscillator with the Hamiltonian

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{m\omega^2 x^2}{2},$$

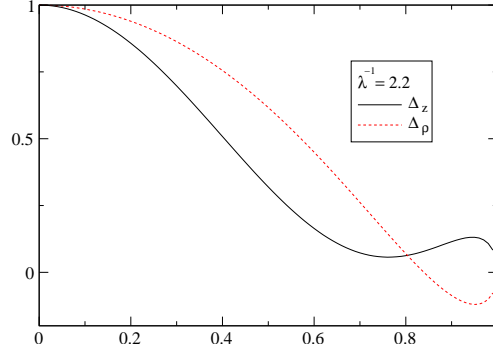


FIG. 3: The order parameter for the aspect ratio $\lambda = 0.45$ (cigarshape trap). The solid line shows $\Delta_0(z, \rho = 0)$, and the dashed line corresponds to $\Delta_0(z = 0, \rho)$.

with the eigenvalues $E_n = \hbar\omega(n + 1/2)$,

$$\varphi_n(x) = \sqrt{\frac{2m\omega}{\pi p_n(x)}} \cos[\Phi_n(x) - \pi/4], \quad (45)$$

where

$$p_n(x) = \sqrt{2m(E_n - m\omega^2 x^2/2)}$$

and

$$\Phi_n(x) = \frac{1}{\hbar} \int_{-\sqrt{2E_n/m\omega}}^x p_n(x') dx' = \frac{\pi}{2}n + \frac{1}{\hbar} \int_0^x p_n(x') dx'.$$

As it was already mentioned, the functions $M_{n_1 n_2}^{(\rho)}(x)$ and $M_{n_1 n_2}^{(z)}(z)$ with n_1 and n_2 close to each other (and both are much larger than unity) are the most important. We therefore introduce $N = (n_1 + n_2)/2$ and $n = n_1 - n_2$, where $n \ll N$, and write the expression for $M_{n_1 n_2}^{(\rho)}(x)$ as (see Eq. (30))

$$\begin{aligned} M_{n_1 n_2}^{(\rho)}(x) &\equiv M_{Nn}^{(\rho)}(x) = \varphi_{N+n/2}(x) \varphi_{N-n/2}(x) \\ &\approx \frac{2m\omega}{\pi p_N(x)} \frac{1}{2} \left\{ \cos[\Phi_{N+n/2}(x) - \Phi_{N-n/2}(x)] + \cos[\Phi_{N+n/2}(x) + \Phi_{N-n/2}(x) - \pi/2] \right\} \\ &\approx \frac{m\omega}{\pi p_N(x)} \left\{ \cos[\Phi_{N+n/2}(x) - \Phi_{N-n/2}(x)] \right\}, \end{aligned}$$

where we neglect the rapidly oscillating contribution $\cos[\Phi_{N+n/2}(x) + \Phi_{N-n/2}(x) - \pi]$. In the case $n \ll N$, the phase difference $\Phi_{N+n/2}(x) - \Phi_{N-n/2}(x)$ can be simplified as

$$\begin{aligned} \Phi_{N+n/2}(x) - \Phi_{N-n/2}(x) &= \frac{\pi}{2}n + \frac{1}{\hbar} \int_0^x [p_{N+n/2}(x') - p_{N-n/2}(x')] dx' \\ &\approx \frac{\pi}{2}n + n\omega \int_0^x \frac{\partial p_N(x')}{\partial E_N} dx' \\ &= n \left[\pi - \arccos \left(x \sqrt{2Nm\omega/\hbar} \right) \right]. \end{aligned} \quad (46)$$

As a result, we obtain the expression

$$\begin{aligned} M_{n_1 n_2}^{(\rho)}(x) &\approx (-1)^n \frac{m\omega}{\pi p_N(x)} \cos \left[n \arccos \left(x \sqrt{2Nm\omega/\hbar} \right) \right] \\ &= (-1)^n T_n \left(x \sqrt{2Nm\omega/\hbar} \right), \end{aligned}$$

which corresponds to Eq. (32) in the dimensional units.

The expression (30) for the function $M_{n_1 n_2}^{(z)}(z)$ involves the first derivative of the wave function. According to the general rules of the WKB approximation (see, e.g., [25]), this derivative should act only on the rapidly varying trigonometric factor in Eq. (45). Therefore, $M_{n_1 n_2}^{(z)}(z)$ can be written in the form

$$\begin{aligned} M_{n_1 n_2}^{(z)}(z) &\equiv M_{Nn}^{(z)}(z) = \frac{1}{2} [\varphi_{N+n/2}(z) \partial_z \varphi_{N-n/2}(z) - \varphi_{N-n/2}(z) \partial_z \varphi_{N+n/2}(z)] \\ &\approx -\frac{m\omega}{\pi\hbar} \left\{ \sin[\Phi_{N+n/2}(z) - \Phi_{N-n/2}(z)] \right\}, \end{aligned}$$

where again we neglect the rapidly oscillating contribution. The application of Eq. (46) gives

$$M_{n_1 n_2}^{(z)}(z) \approx (-1)^n \frac{m\omega}{\pi\hbar} \sin \left[n \arccos \left(x \sqrt{2Nm\omega/\hbar} \right) \right],$$

and, assuming $n > 0$, we obtain

$$M_{n_1 n_2}^{(z)}(z) \approx (-1)^n \frac{m\omega}{\pi\hbar} \sqrt{1 - \left(x \sqrt{2Nm\omega/\hbar} \right)^2} U_{n-1} \left(x \sqrt{2Nm\omega/\hbar} \right).$$

Note also, that one obtains the same result if, instead of differentiating the WKB wave function, Eq. (45), one uses the well-known relations between the wave functions of a harmonic oscillator and its derivatives.

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